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SOLIDS FRICTION FACTOR CORRELATION FOR VERTICAL UPWARD PNEUMATIC CONVEYINGS

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Abstract—A new correlation for solids friction factor in vertical upward pneumatic conveying lines is hereby proposed. This correlation consists of easy-to-measure variables which makes it quite practical. Its accuracy is about ∓20%. The proposed correlation can be used in conjunction with the theoretically derived equations suggested earlier (Özbelge 1983) to calculate voidage, drag velocity, solids phase velocity and density without the need of any pressure drop data.

1. INTRODUCTION

Design of a pneumatic transport system is an art rather than a science. For such a complex system, a macroscopic model with some simplifying assumptions has been developed previously by Özbelge (1983) to calculate average solids phase velocity and density, voidage, the external force acting on the solids phase, the relative or drag velocity between the phases. Total pressure drop data was necessary to use this theoretical model.

Numerous correlations for predicting pressure drop in dilute phase vertical pneumatic conveying as reviewed by Leung & Wiles (1976) are available in the literature; but still no satisfactory method exists for calculating two-phase pressure drop.

Therefore, the aim of this paper is to present a correlation for solids friction factor and a procedure which makes use of the above-mentioned correlation together with the previously derived theoretical model and its conclusions by Özbelge (1983), thus removing the need for any pressure drop data to calculate voidage, drag velocity, solids friction factor, solids phase velocity and density.

2. DETERMINATION OF SOLIDS FRICTION FACTOR

The solids friction factor arises due to the solid particle-pipe wall interactions. Three ways of solution can be used to calculate the solids friction factor;

(a) from external force on solids phase: the total external force, F_k, within a pipe of length "L" has been obtained by Özbelge (1983) as

$$F_k = F_{e,s}L = f_{e,s}\frac{\pi R_T^2 \rho_{ds} L}{\rho_p V_p} \quad [1]$$

where $F_{e,s}$ = total external force acting on the whole solids phase per unit length of test section; $f_{e,s}$ = external force per solid particle; R_T = pipe radius; ρ_{ds} = dispersed solids density; ρ_p = material density of solid particles; V_p = volume of a spherical particle; L = length of test section. $f_{e,s}$ in [1] is given by Özbelge (1983) as;

$$f_{e,s} = \frac{1}{2}A_p\rho_G < u_r >^2 C_D - g\rho_p V_p \quad [2]$$

where A_p = projected area of a spherical solid particle with a radius R_p, πR_p^2; ρ_G = density of gas; u_r = relative velocity between the phases; C_D = drag coefficient for a single particle in an assemblage of particles of voidage α; g = acceleration of gravity.

Considering that each phase is a continuum, the definition of friction factor given by Bird *et al.* (1960) can be applied to the solids phase to obtain the solids friction factor as follows

$$F_k = AK_e f_s \tag{3}$$

where A = a characteristic area, $2\pi R_T L$; K_e = a characteristic kinetic energy for the solids phase, $\frac{1}{2}\rho_{ds}u_{ds}^2$; f_s = solids friction factor; u_{ds} = solids phase velocity. Equation [3] is equivalent to

$$F_k = (2\pi R_T L)(\tfrac{1}{2}\rho_{ds}u_{ds}^2)f_s. \tag{4}$$

From [1] and [4], f_s can be solved as

$$f_s = \frac{3f_{e,s}R_T}{4\pi R_p^3 \rho_p u_{ds}^2} \tag{5}$$

(b) from pressure drop due to solids friction, ΔP_{fs}: Hariu & Molstad (1949), Konno & Saito (1969), Capes & Nakamura (1973), and Yousfi & Gau (1974) defined the solids friction factor following the Fanning equation based on solids phase velocity and the dispersed solids density,

$$f_s = \frac{\Delta P_{fs} D_T}{2Lu_{ds}^2\rho_{ds}} \tag{6}$$

where ΔP_{fs} = pressure drop due to solids friction only; D_T = pipe diameter.

(c) from drag velocity and terminal velocity:

The equation of motion for the solids phase is:

$$Nm_p\frac{\mathrm{d}u_{ds}}{\mathrm{d}t} = NA_pC_D\rho_G(u_f - u_{ds})^2/2 - Nm_pg - \Delta P_{fs}A_c \tag{7}$$

where N = number of particles in a length L of the pipe; m_p = mass of a single particle; t = time; u_f = fluid phase velocity; A_c = cross-sectional area of the pipe.

Interparticle collisions and collisions with the pipe wall retard the particles; this retarding force, $\Delta P_{fs}A_c$, is

$$\Delta P_{fs}A_c = \frac{2f_sLu_{ds}^2\rho_{ds}}{D_T}A_c. \tag{8}$$

The dispersed solids density, ρ_{ds}, is equal to

$$\rho_{ds} = \frac{Nm_p}{A_cL}. \tag{9}$$

Substituting [8] and [9] into [7] and considering the fully developed steady flow case, $\mathrm{d}u_{ds}/\mathrm{d}t = 0$, it follows that:

$$C_DA_p\rho_G(u_f - u_{ds})^2/2 - m_pg - \frac{2f_sm_pu_{ds}^2}{D_T} = 0. \tag{10}$$

While a single particle is being transported upwards in an unbounded gas stream, when the particle has no acceleration, the drag force on the particle is equal to the gravitational force,

$$C_{D_t}A_p\rho_G(u_f-u_{ds})^2/2 = m_p g. \qquad [11].$$

In this case, the relative velocity is equal to the particle terminal velocity. Therefore, from [11]

$$u_t = u_f - u_{ds} = (2m_p g/C_{D_t}A_p\rho_G)^{0.5} \qquad [12]$$

where u_t = terminal velocity of a particle; C_{D_t} = drag coefficient for a single particle in an unbounded air stream. Substituting for $A_p\rho_G$ from [12] into [10] and after simplifying,

$$C_D/C_{D_t}[(u_f-u_{ds})/u_t]^2 = 1+\frac{2f_s u_{ds}^2}{gD_T} \qquad [13]$$

Equation [13] is obtained, where $u_f - u_{ds} = u_r$, average drag velocity between the phases. C_D, C_{Dt} and u_r in [13] are calculated as explained by Özbelge (1983) to be able to solve f_s from [13].

Calculated f_s values have been correlated according to the Marquardt's algorithm (1963) which is a technique to fit the data to a nonlinear mathematical model by nonlinear least squares computations.

3. SOLIDS FRICTION FACTOR CORRELATIONS

Yang (1974, 1978) obtained correlations for solids friction factor in vertical pneumatic conveying lines including the voidage, α, explicitly. The general form of these correlations was

$$f_p\frac{\alpha^3}{(1-\alpha)} = a\left[(1-\alpha)\frac{u_t}{u_r}\right]^{-b} \qquad [14]$$

where a and b = the regression constants varying with the different sets of experimental data; f_p = solids friction factor calculated according to the following equation

$$f_p = \frac{2D_T\Delta P_{fs}}{Lu_{ds}^2\rho_{ds}}. \qquad [15]$$

Because of the different definition in Fanning equation [15], f_p values used by Yang (1974, 1978) are equal to four times f_s values in this work. ΔP_{fs} data of Hariu & Molstad (1949) have been used by Yang (1974, 1978) to calculate f_p values from [15] which have later been correlated to obtain the constants a and b as 0.0206 and 0.869, respectively. Yang (1974) reported that [14] was good to $\mp 30\%$ for almost 90% of the data by Hariu & Molstad (1949).

In this study, f_s values calculated from [5] and [13], using the data of Hariu & Molstad (1949), are the same; but they are different than those calculated from [6]. This discrepancy may arise from the error included in ΔP_{fs} values due to the difficulty of isolating the pressure drop caused by solids friction from the pressure drops caused by gas flow and static head of solids. The proposed correlation of this study is;

$$f_s = 0.0054\left(\frac{W_r\rho_G}{\rho_p}\right)^{-0.115}\left(\frac{u_G D_p}{u_r D_T}\right)^{0.339} \qquad [16]$$

where u_G = superficial gas velocity; D_T = pipe diameter; D_p = particle diameter; W_r = solids loading ratio (SLR). SLR can be written in the following way,

$$W_r = G_{ds}/G_G \qquad [17]$$

where $G_{ds} = u_{ds}\rho_{ds} = u_{ds}\rho_p(1-\alpha)$, mass flux of solids; $G_G = u_G\rho_G$, mass flux of gas. Therefore, W_r is equal to,

$$W_r = \frac{u_{ds}\rho_p(1-\alpha)}{u_G\rho_G}. \qquad [18]$$

4. DISCUSSION

Results of the previous work by Özbelge (1983) prove that for the same gas velocity and the same particle size, the average drag velocity and the external force on solids are independent of solids loading ratio, W_r, for dilute upward flowing gas–solids suspensions; u_r and $f_{e,s}$ increase with the increasing u_G and D_p (figures 1 and 2). This, in turn, makes possible to express[16], the correlation for solids friction factor, in terms of u_G, ρ_G, ρ_p, u_r rather than u_f, ρ_f, ρ_{ds}, u_{ds} which are strong functions of α. In such a correlation, the effects of α and u_{ds} are included by the presence of the variable W_r which is much easier to measure than the voidage, α.

The second dimensionless group in[16] can be equivalently written in two different forms, one in terms of gas and particle Froude numbers, and the other one in terms of gas and particle Reynolds numbers as follows;

$$\frac{u_G D_p}{u_r D_T} = \left(\frac{Fr_G D_p}{Fr_p D_T}\right)^{1/2} \qquad [19]$$

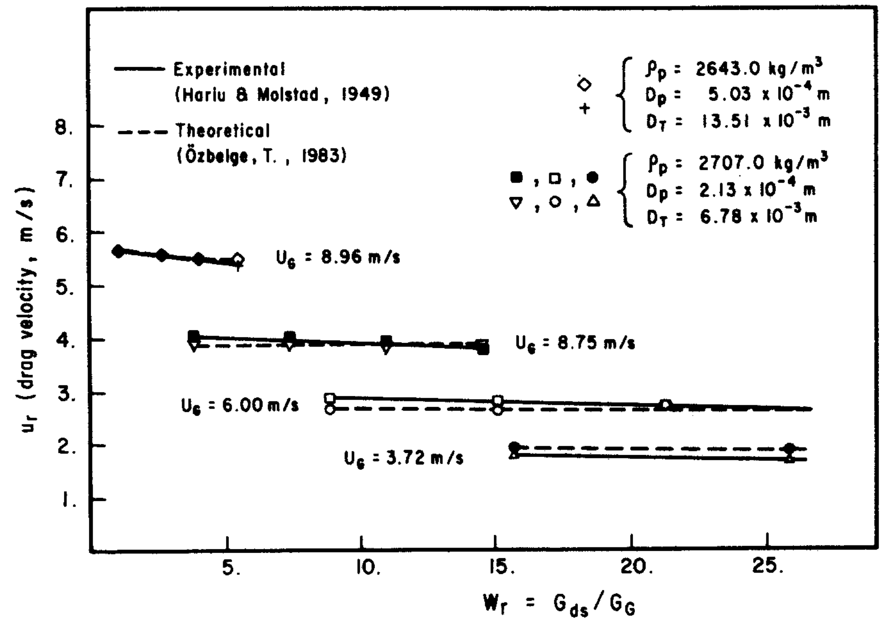
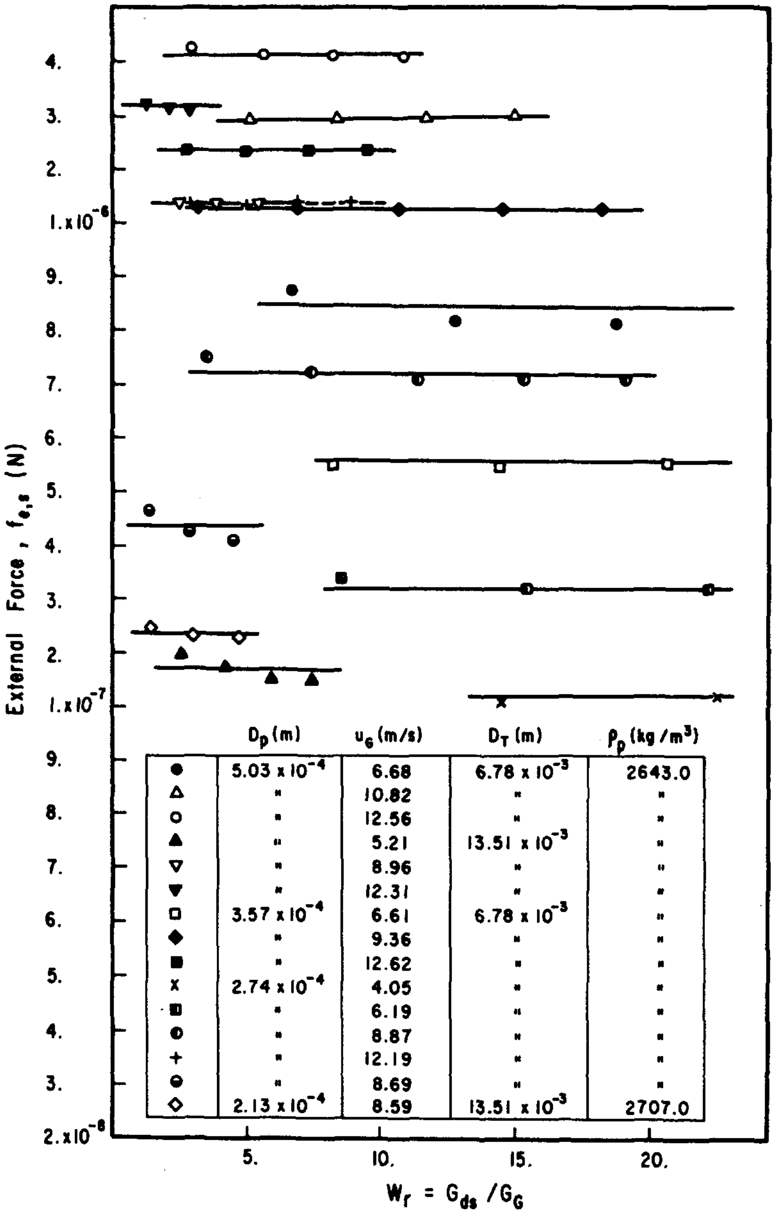


Figure 1. Dependence of drag velocity on solids loading ratio, superficial gas velocity, and particle size.



	D_p (m)	u_G (m/s)	D_T (m)	ρ_p (kg/m^3)
●	5.03 x 10^{-4}	6.68	6.78 x 10^{-3}	2643.0
△	"	10.82	"	"
○	"	12.56	"	"
▲	"	5.21	13.51 x 10^{-3}	"
▽	"	8.96	"	"
▼	"	12.31	"	"
□	3.57 x 10^{-4}	6.61	6.78 x 10^{-3}	"
◆	"	9.36	"	"
■	"	12.62	"	"
x	2.74 x 10^{-4}	4.05	"	"
◨	"	6.19	"	"
◐	"	8.87	"	"
+	"	12.19	"	"
⊖	"	8.69	"	"
◇	2.13 x 10^{-4}	8.59	13.51 x 10^{-3}	2707.0

Figure 2. Dependence of external force on solids loading ratio, superficial gas velocity, particle size and pipe diameter [calculated by Özbelge (1983) using experimental data of Hariu & Molstad (1949)].

where, $\mathrm{Fr}_G = u_G^2/gD_T$; $\mathrm{Fr}_p = u_r^2/gD_p$ and

$$\frac{u_G D_p}{u_r D_T} = \frac{\mathrm{Re}_G}{\mathrm{Re}_p}\left(\frac{D_p}{D_T}\right)^2 \qquad [20]$$

where $\mathrm{Re}_G = u_G D_T \rho_G/\mu_G$; $\mathrm{Re}_p = u_r D_p \rho_G/\mu_G$.

Equation [16] is good to $\mp 20\%$ for 98% of the data by Hariu & Molstad (1949). The comparison covers experimental data points with mean particle diameter ranging from 213 to 503 μm, particle density from 2600 to 2700 kg/m^3, voidage from 0.980 to 0.999, pipe diameter from 6.76×10^{-3} to 13.54×10^{-3} m, superficial gas velocity from approx. 6.0 to 12.5 m/s. ρ_G for air is taken as 1.185 kg/m^3.

The confidence limits of the parameters were found as 0.0054 ∓ 0.0014, -0.115 ∓ 0.030, and 0.339 ∓ 0.109, respectively.

The important feature of [16] is that it removes the need for any pressure drop data to calculate drag velocity, voidage, solids phase velocity, dispersed solids density and solids friction factor. The devised trial and error procedure can be described in five steps:

(a) Drag velocity is obtained from [16] for an estimated value of f_s, if the physical and the geometrical properties of the system (ρ_p, ρ_G, D_p, D_T), superficial gas velocity, u_G, and the solids feed rate (kg/s), so the solids loading ratio, W_r, are known.

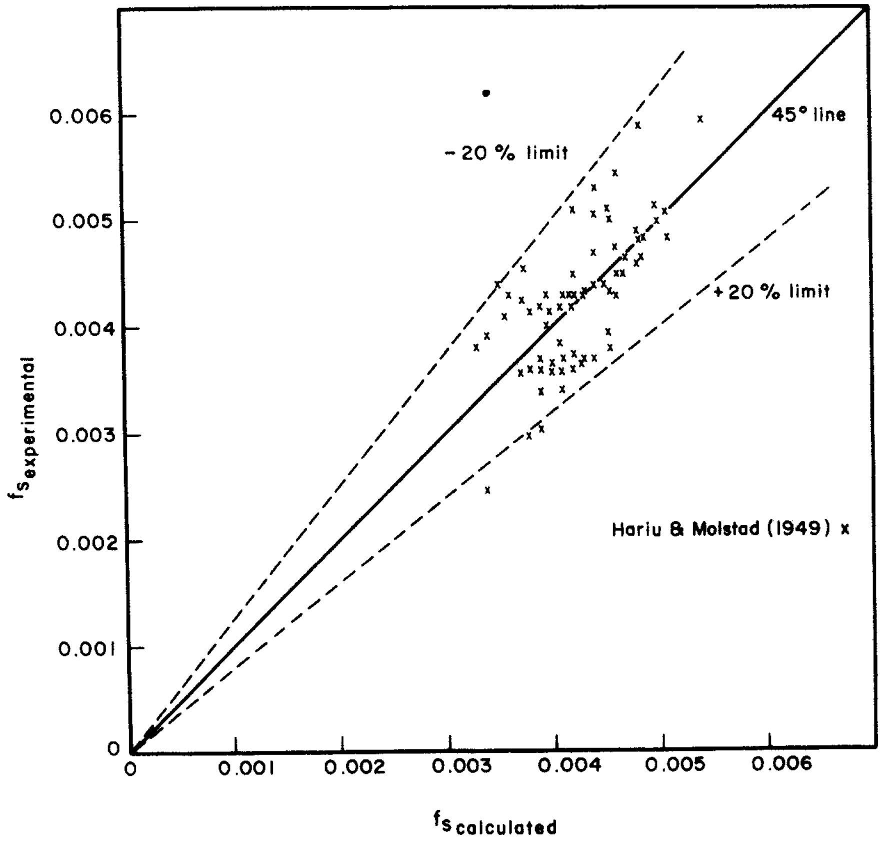
(b) External force on a solid particle, $f_{e,s}$, is calculated from [2]. For lightly loaded systems, C_D approaches to C_{D_t} which can be obtained from standard charts as a function of particle Reynolds number (Bird *et al.* 1960). Since the proposed correlation is valid for α from 0.980 to 0.999, according to the relationship given by Wen & Galli (1971), C_{D_t} is multiplied by an average correction factor of 1.05 to get C_D.

(c) Solids phase velocity, u_{ds}, is predicted from [5] for an estimated value of f_s in step (a) and accordingly calculated value of $f_{e,s}$ in step (b).

(d) Fluid phase velocity, then the voidage are calculated respectively from the following equations,

$$u_f - u_{ds} = u_r \qquad [21]$$

$$\alpha = u_G/u_f \qquad [22]$$



$$f_s = 0.0054 \left(\frac{W_r \, \rho_G}{\rho_p}\right)^{-0.115} \left(\frac{u_G}{u_r} \, \frac{D_p}{D_T}\right)^{0.339}$$

Figure 3. Comparison of measured and predicted solids friction factors [f_s calculated using experimental data of Hariu & Molstad (1949) vs f_s calculated with [16]].

(e) From [18], an α value can be calculated corresponding to the solids loading ratio, W_r, chosen in step (a) and to the u_{ds} value from step (c); trial and error procedure between steps (a) and (e) is repeated until the α value obtained in step (d) is the same as that calculated in step (e).

5. CONCLUSIONS

Solids phase velocity and density, voidage, drag velocity, solids friction factor in vertical pneumatic conveying lines were predicted from a new correlation for solids friction factor in conjunction with the theoretical derivations suggested earlier by Özbelge (1983) by a devised trial and error procedure.

The proposed correlation is valid for the following ranges of the variables: voidage, α, 0.980–0.999; mean particle diameter, D_p, 213–503 μm; particle density, ρ_p, 2600–2707 kg/m^3; pipe diameter, D_T, 6.76×10^{-3}–13.54×10^{-3} m; superficial gas velocity, u_G, 6.0–12.5 m/s. The carrier gas is air. The correlation is good to $\mp 20\%$ if experimental data of Hariu & Molstad (1949) are used. Figure 3 shows the scattering of f_s values from experimental data and f_s values from [16] around the 45° line.

The obtained correlation for solids friction factor is quite practical; because it does not have voidage term explicitly, although it includes its effect. Therefore, it should be worthwhile to test for the extension of its range of applicability with the other experimental data.

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